Market Microstructure in the Big-data Era: Improving High-frequency Price Prediction via Machine Learning

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Outline

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Price discovery

- Price discovery is key in microstructure and financial markets
- Fundamental question: where does information originate?
  - From which trading venues (NYSE vs Nasdaq)?
  - Via what order types (trades vs quotes, top-of-book vs depth-of-book)?
  - Via which asset classes (spot vs derivatives)?
- The existing empirical microstructure literature typically uses vector autoregressive (VAR) or vector error correction (VEC) models. They share the following features:
  - Structural functional forms with a relatively small set of features
  - In-sample attribution of the information shares

Challenges in the machine age

- Trading in the machine age (e.g., in the US equities market)
  - Extremely fast: algorithmic and high-frequency trading; 20% of trades arrive in $<$ 1ms clusters (Menkveld, 2018)
  - A highly fragmented market: 16 public exchanges, internalization, dark pools
  - Voluminous trading data: level-3 order book messages
- Challenges:
  - A much-expanded feature set (based on full order books from many markets)
  - Complex non-linear effects and feature interactions (from algorithmic trading strategies such as dice-and-slice, pinging, layering, cross-market/cross-asset arbitrages...)
- Need for machine learning (ML) models

Paper in a nutshell

- Propose machine learning (ML) models suitable for empirical market microstructure with big data challenges
- Apply it to price discovery analysis
  - Which exchange contributes the most to price discovery?
  - Which part of the data feed contributes the most to price discovery?
- Key takeaways:
  - ML models designed for processing sequential data, long short-term memory (LSTM), and Transformers, perform much better in predicting short-term midquote returns
  - Nasdaq contributes the most to price discovery. Dropping its data feeds leads to about 6% drop in out-of-sample R^2
  - Data feeds beyond the top-of-book are informative. Dropping them leads to about 13% drop in out-of-sample R^2

Literature

- Empirical market microstructure on price discovery
  - Models: VAR (Hasbrouck, 1991); "information share" via VECM (Hasbrouck, 1995); "component share" (Harris, McInish, and Wood, 2002); "information leadership share" (Putniņš, 2013); VAR + VECM (Hagströmer and Menkveld, 2023);
  - Applications: spot vs futures (Hasbrouck, 2003); cross-border listings (Eun and Sabherwal, 2003); dark vs lit trading (Hendershott and Jones, 2005)
- Financial machine learning
  - Cross-section asset pricing (Gu, Kelly, and Xiu, 2020); Mutual funds selection (Li and Rossi, 2020); Robot-advising (Rossi and Utkus, 2020); Corporate bond return prediction (Bali et al., 2020)
- *Our contribution*: apply ML models suitable for empirical microstructure

Outline

Data

- "Direct feeds" from public exchanges
  - Level 3 order-book messages: all add (new limit orders), cancel/modification of existing orders, and trade messages
  - Timestamped to microsecond precision
- 30 constituent stocks of the Dow Jones Index (DJI). 54 trading days interspersed from the year of 2017 to 2021.
- For each exchange, we build the entire order book based on the direct feed messages

Limit order book (LOB) market

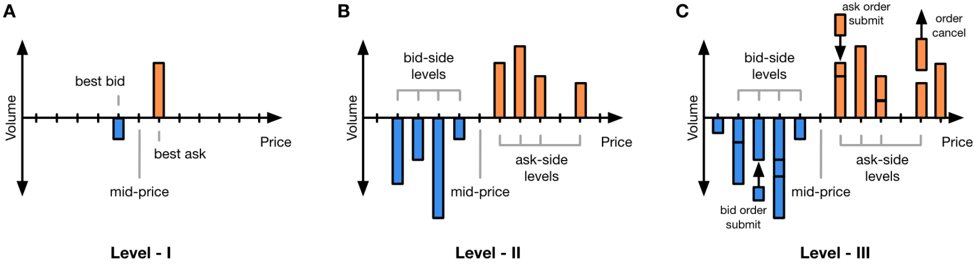
- Most liquid markets use limit order books (LOBs) for trading
- A limit order book is essentially a collection of unexecuted quotes
  - Each quote specifies the price and quantity the trader is willing to trade
  - New quotes can be continuously added and existing quotes can be canceled, modified, or executed against incoming marketable orders

	Shares	Price
ASKS	249	172.36
	560	172.35
	349	172.34
	525	172.33
	125	**172.32**
BIDS	100	**172.31**
	323	172.30
	449	172.29
	364	172.28
	249	172.27

LOB data types

- Level-I: the best bid/ask prices and volumes,
- Level-II: price and aggregated volume across a certain number of price levels
- Level-III: non-aggregated orders placed by market participants

Figure 1: LOB data types. Source: Wu et al. (2022)



Direct feeds

- "Direct feeds" examples

(a) Add message

datetime	mtype	micros	seq	delta	source	symbol	oid	size	price	side	flags
2014-07-15 14:01:00.465239	add	50460465239	254208711	60	INET	AAPL	193553906	59	95.46	S	-

(b) Cancel/modification message

datetime	mtype	micros	seq	delta	source	symbol	oid	size
2014-07-17 13:01:00.976771	mod	46860976771	210279229	43	INET	MSFT	162545830	0

(c) Trade message

datetime	mtype	micros	seq	delta	source	symbol	oid	size	price	side	flags
2014-08-12 14:02:35.414432	trd	50555414432	219933620	102	INET	YHOO	167243090	100	35.290	B	-

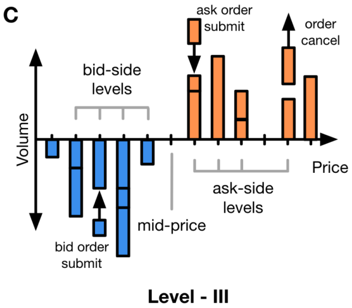
Figure 2: Direct feeds

Outline

LOB actions

- LOB is constantly changing due to addition, modification, and execution of orders
  - New quotes can be continuously added, and existing quotes can be canceled, modified, or executed against incoming marketable orders
  - At different price levels

Figure 3: LOB actions. Source: Wu et al. (2022)



Selected Features

- **LOB events** and their lagged values (50 lags), *from each exchange*
  - **Trade-BBO-Changing**: Executions moving BBO
  - **Trade-NonBBO-Changing**: Execution not moving BBO
  - **Add-BBO-Improving**: Add orders improving BBO
  - **Cancel-BBO-Worsening**: Cancel orders worsening BBO
  - **Add-at-BBO**: Add orders adding depth at the current BBO
  - **Cancel-at-BBO**: Cancel orders removing depth at the current BBO
  - **Add-<=5lvlBBO**: Add orders adding depth <= 5 levels from BBO
  - **Cancel-<=5lvlBBO**: Cancel orders removing depth <= 5 levels from BBO
  - **Add->5lvl-BBO**: Add orders adding depth > 5 levels from BBO
  - **Cancel->5lvl-BBO**: Cancel orders removing depth > 5 levels from BBO
- **Midquote return**, and their lagged values (50 lags), *from each exchange*

Example: suppose a new limit order adding 300 shares at the best bid, then the variable "Add-at-BBO" takes the value of 300, and all other variables are 0

Target and performance evaluation

- Target
  - Short-term midquote return (over the next five events)
  - Clock runs in event time
- Performance evaluation
  - Out-of-sample R^2:

$$R^2(\mathrm{Y}, \widehat{\mathrm{Y}}) = 1 - \frac{\sum_i \left(Y_i - \widehat{Y}_i\right)^2}{\sum_i \left(Y_i - \frac{1}{n}\sum_i Y_i\right)^2} \tag{1}$$

  - $R^2 > 0$: the model outperforms the out-of-sample mean

Machine learning models

- Simple linear model (OLS)
- Linear models with penalties
  - Elastic net penalties (Elastic Net)
- Tree-based models
  - Random forests (RF)
  - Gradient boosted regression trees (GBRT)
- Artificial neural networks
  - Feedforward or multiplayer perceptron (MLP)
  - Long short-term memory (LSTM)
  - Transformer

Linear model with penalties

- Too many features might lead to overfitting
- One solution is to add penalties to the loss function
- Elastic net penalties: penalize + shrink

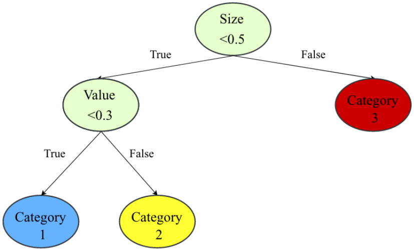
$$\min_{w} \frac{1}{2n_{\text{samples}}} \| \overbrace{X}^{\text{features}} w - y\|_2^2 + \alpha\rho \overbrace{\|w\|_1}^{\text{Lasso}} + \frac{\alpha(1-\rho)}{2} \overbrace{\|w\|_2^2}^{\text{Ridge}} \tag{2}$$

- Solves overfitting but is still linear.

Random forests and boosted regression trees

- Regression tree

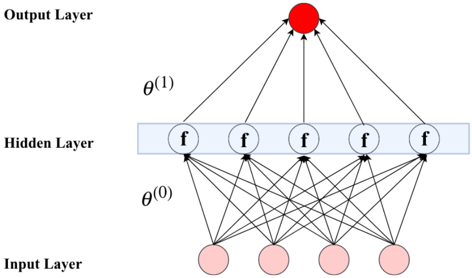
Figure 4: Tree example. Source: Gu, Kelly, and Xiu (2020)



- Tree splitting captures non-linearities and flexible interactions
- Both random forests and boosted regression trees are ensemble methods
- Combine base estimators to improve generalizability/robustness over a single estimator.

Feedforward networks

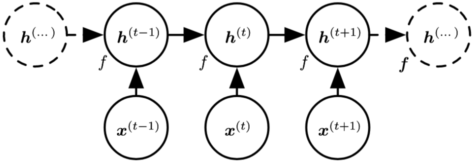
- Feedforward networks



- Its non-linear activation function captures non-linearities and flexible interactions
- However, it is not designed for processing temporal sequence

Long short-term memory (LSTM)

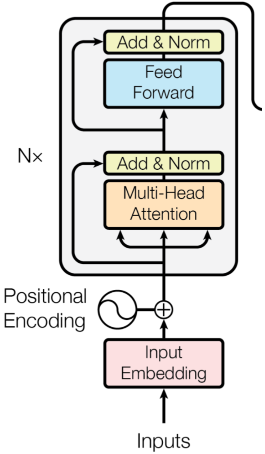
- Recurrent neural network (RNN) models are designed to process sequential data like time series



- Long short-term memory (LSTM) is a gated RNN model that addresses the vanishing gradient problem
- Uses a series of gate functions to control information flow
- Captures long-term temporal dependence

Transformer

- Uses multi-head attention mechanism
- More attention, i.e., weights, given to more important temporal information
- The whole sequence is attended and no loss in temporal information



Tranining, validation, and testing sample split

- We split each trading day into 13 half-an-hour intervals
- Training (system parameters fitting), validation (hyper/tuning parameters fitting), and testing based on inter-day rolling windows. For example,
  - 09:30 - 10:00 today as training; 09:30 - 10:00 tomorrow as validation; 09:30 - 10:00 the day after as testing
- We consider the following hyperparameters:
  - Elastic-net: $\rho = 0.5$; $\alpha = (0.1, 0.01, 0.001, 0.0001)$
  - RF: Depth = (2, 4, 6); #Trees = 300; #Features in each split = (3, 5, 10)
  - BRT: Depth = (1, 2); #Trees = (100, 1000); Learning rate = (0.01, 0.1)
  - MLP: units = ((32, 16), (32))
  - LSTM: LSTM units = ((32, 16), (32)); MLP units = ((32, 16), (32))
  - Transformer: # Attention head = (2, 4); Key dimension = (16, 32)

Outline

Introduction

Data

Methodology

Results

Conclusion

Prediction results

- LSTM and Transformers consistently outperform other ML models
- They capture long-term temporal dependence in the feature time series which can result from algorithmic trading strategies
  - E.g., informed traders slice and dice their orders to minimize price impact

Table 1: MSE and out-of-sample R^2. Averages across all tickers and intraday intervals.

Model	MSE-Val	MSE-Test	R^2_{OOS}
OLS	1.0612	1.0657	-0.0657
Elastic Net	0.9784	0.9815	0.0185
RF	0.9974	0.9975	0.0024
GBRT	0.9906	0.9914	0.0086
MLP	0.9958	0.9959	0.0041
LSTM	0.8879	0.8961	**0.1039**
Transformers	0.8426	0.8509	**0.1491**

Permutation importance

- To assess the importance of a feature or several features, permutate (randomly shuffle the ordering) them in the testing set
- Then compare the change in out-of-sample R^2
- Different from in-sample feature importance
- Agnostic to model choice
- We use the best-performing model, **Transformers**, for the permutation importance calculations

Permutation importance (exchange)

- Which exchange contributes the most to price discovery?
- Look at the R^2 drops when an exchange's data feed is permutated
- Nasdaq's data feed is relatively most important. But the drop in R^2 is mild in absolute magnitude.

Table 2: R^2 of permutated testing samples. The first line shows the R^2 of the original sample. The second lines and so on report the R^2 changes when an exchange's data feed is permutated.

Feature Type	Feature	MSE	R^2_{OOS}	Drop in R^2_{OOS} (%)
All	All	0.8492	0.1508	0.0
Exchange	ARCA	0.8509	0.1491	1.14
	BATS	0.8516	0.1484	1.61
	EDGX	0.8512	0.1488	1.33
	INET	0.8583	0.1417	**6.01**
	NYSE	0.8511	0.1489	1.26

Permutation importance (LOB levels)

- Which part of the limit order book (e.g., beyond the best five levels, or within the best five levels) contributes the most to price discovery?
- Look at the R^2 drop when a different part of the data feeds is permutated
- Data feeds beyond the five best levels have limited information; within five levels much more important

Table 3: R^2 of permutated testing samples. The first line shows the R^2 of the original sample. The second lines and so on report the R^2 changes when part of the data feed is permutated.

Feature Type	Feature	MSE	R^2_{OOS}	Drop in R^2_{OOS} (%)
All	All	0.8492	0.1508	0.0
LOB Level	Beyond five best levels	0.8568	0.1432	5.03
	Beyond top-of-book	0.8693	0.1307	**13.36**

Outline

Conclusion

- Machine learning (ML) models, such as long short-term memory (LSTM) and Transformers, designed for processing sequential data have improved prediction performance for LOB midquote returns than other ML models
- Capture long-term temporal dependence in the feature time series which is needed for analyzing trade in the machine age
- In terms of price discovery analysis:
  - Nasdaq's trading contains most information relative to other exchanges. Dropping its data feeds leads to about 6% drop in out-of-sample R^2
  - Data feeds beyond the top-of-book are informative. Dropping them leads to about 13% drop in out-of-sample R^2
- Future extensions:
  - Include other potential features such as order queuing information
  - More direct evidence of the importance of long-term dependence